Glue content and mixing angle of the $\eta - \eta'$ system. The effect of the isoscalar 0^- continuum.

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Abstract

Masses and topological charges of the η and η' mesons are expressed in terms of the singlet-octet mixing angle θ. Contributions of the pseudoscalar 0^- continuum are evaluated in a model independant way. Applications to the decay $\eta \to 3\pi$ and to the radiative decay of vector mesons involving η and η' are considered. Agreement with experiment is in general good and the results quite stable for $-30.5° \lesssim \theta \lesssim -18.5°$.

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I. INTRODUCTION

The subject of $\eta - \eta$ mixing has been a topic of discussion from the time $SU(3)$ flavour symmetry was proposed.The gluon axial anomaly and the corresponding topoligical charges of the isoscalar mesons imply that the $SU(3)$ singlet axial-vector current is not conserved in the chiral limit. Traditionally,analyses based on the Gell-Mann-Okubo mass formula led to the adoption of a small value for the octet-singlet mixing angle, $\theta \simeq -10°$. Subsequent study of the axial-anomaly generated decays η, $\eta' \to 2\gamma$ and measurement of the decay rates led to adoption of larger values for the mixing angle, $\theta = -25°$ - $-20°$ [1]. Phenomenological applications of the mixing and of the anomalies were considered [2] [3]. In ref. [4] the PCAC corrections to the calculation of the decay rates of the η and η' to two photons, i.e the corrections to the soft meson limits arising from the large masses of the isoscalars were evaluated and found to be quite large.

It is the purpose of the present work is to express the mases and topological charges of the η and η' mesons in terms of the mixing angle θ.Contributions of the pseudoscalar 0^- continuum will be taken care of in a model independant way using only known QCD expressions of spectral functions in the deep euclidean region and the fact that the main contribution to dispersion integrals which enter in the evaluation of $SU(3)$x$SU(3)$ breaking arise from the energy interval $1.5\text{GeV}^2 - 2.5\text{GeV}^2$ where the isoscalar pseudoscalar resonances lie.

In section II the quark and gluon content of the η and η' are related to the mixing angle θ. In section III the decay rate $\Gamma\left(\eta \to 3\pi\right)$ is related to the quark mass ratio $\frac{m_u}{m_d}$ and values of the latter infered.In section IV the analysis of Ball et al. [2] of the decays $P \to V\gamma$ is modified to take chiral symmetry breaking into account.

Agreement with experiment is in general good and the results are quite insensitive to exact values of the mixing angle θ.

II. MASSES OF THE η AND η'

The isoscalar components of the octet of axial-vector currents couple to the physical η and η' mesons

$$\langle 0 | A_\mu^{(8)} | \eta(p) \rangle = 2i f_8 \cos\theta p_\mu$$
$$\langle 0 | A_\mu^{(0)} | \eta(p) \rangle = -2i f_0 \sin\theta p_\mu \tag{2.1}$$
$$\langle 0 | A_\mu^{(8)} | \eta'(p) \rangle = 2i f_8 \sin\theta p_\mu$$
$$\langle 0 | A_\mu^{(0)} | \eta'(p) \rangle = 2i f_0 \cos\theta p_\mu$$

θ is the singlet-octet mixing angle. In the $SU(3)$ limit $f_8 = f_\pi = 92.4\text{MeV}$. The axial-vector currents are given in terms of the quark fields

$$A_\mu^{(8)} = \frac{1}{\sqrt{3}} \left(\bar{u}\, \gamma_\mu\gamma_5 u + \bar{d}\, \gamma_\mu\gamma_5 d - 2\, \bar{s}\, \gamma_\mu\gamma_5 s \right)$$
$$A_\mu^{(0)} = \sqrt{\frac{2}{3}} \left(\bar{u}\, \gamma_\mu\gamma_5 u + \bar{d}\, \gamma_\mu\gamma_5 d + \bar{s}\, \gamma_\mu\gamma_5 s \right) \tag{2.2}$$

Unlike f_π, f_0 and f_8 are not related to any physical process. The currents which project on the physical η and η' states are, respectively

$$A_\mu^{(\eta)} = \left(\frac{A_\mu^{(8)} \cos\theta}{f_8} - \frac{A_\mu^{(0)} \sin\theta}{f_0} \right)$$
$$A_\mu^{(\eta')} = \left(\frac{A_\mu^{(8)} \sin\theta}{f_8} + \frac{A_\mu^{(0)} \cos\theta}{f_0} \right) \tag{2.3}$$

i.e.

$$\langle 0 | A_\mu^{(\eta)} | \eta(p) \rangle = 2i p_\mu \quad , \quad \langle 0 | A_\mu^{(\eta)} | \eta'(p) \rangle = 0 \tag{2.4}$$

and

$$\langle 0 | A_\mu^{(\eta')} | \eta(p) \rangle = 0 \quad , \quad \langle 0 | A_\mu^{(\eta')} | \eta'(p) \rangle = 2i p_\mu \tag{2.5}$$

When the divergence of the currents is taken, the singlet component picks up a gluon anomaly term

$$\partial_\mu A_\mu^{(8)} = \frac{2}{\sqrt{3}} \left(m_u\, \bar{u}\, i\gamma_5 u + m_d\, \bar{d}\, i\gamma_5 d - 2m_s\, \bar{s}\, i\gamma_5 s \right)$$

$$\partial_\mu A_\mu^{(0)} = 2\sqrt{\frac{2}{3}} \left(m_u\, \bar{u}\, i\gamma_5 u + m_d\, \bar{d}\, i\gamma_5 d + m_s\, \bar{s}\, i\gamma_5 s \right) + \sqrt{\frac{2}{3}}\frac{3\alpha_s}{4\pi}G\widetilde{G} \tag{2.6}$$

With $G\widetilde{G} = G_{\mu\nu}\widetilde{G}^{\mu\nu}$, $G_{\mu\nu}$ being the gluonic field strength tensor and $\widetilde{G}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}G^{\rho\sigma}$ it's dual. As $m_{u,d} \ll m_s$,

$$\partial_\mu A_\mu^{(8)} \simeq \frac{-4}{\sqrt{3}}m_s\, \bar{s}\, i\gamma_5 s \tag{2.7}$$

$$\partial_\mu A_\mu^{(0)} \simeq 2\sqrt{\frac{2}{3}}m_s\, \bar{s}\, i\gamma_5 s + \sqrt{\frac{2}{3}}\frac{3\alpha_s}{4\pi}G\widetilde{G} \tag{2.8}$$

constitute good approximations. There results for the masses of the η and η'

$$m_\eta^2 = \frac{1}{\sqrt{3}f_\pi}\left(\frac{\cos\theta}{F_8} + \frac{\sin\theta}{\sqrt{2}f_0}\right)\langle 0\,|-2im_s\, \bar{s}\, \gamma_5 s|\,\eta\rangle - \frac{\sin\theta}{\sqrt{6}f_\pi F_0}\langle 0\left|\frac{3\alpha_s}{4\pi}G\widetilde{G}\right|\eta\rangle \tag{2.9}$$

$$m_{\eta'}^2 = \frac{1}{\sqrt{3}f_\pi}\left(\frac{\sin\theta}{F_8} - \frac{\cos\theta}{\sqrt{2}f_0}\right)\langle 0\,|-2im_s\, \bar{s}\, \gamma_5 s|\,\eta'\rangle + \frac{\cos\theta}{\sqrt{6}f_\pi F_0}\langle 0\left|\frac{3\alpha_s}{4\pi}G\widetilde{G}\right|\eta'\rangle \tag{2.10}$$

with the notation $f_{0,8} = F_{0,8}f_\pi$. Using eq.(2.5) one has for the topological charges

$$\langle 0\left|\frac{3\alpha_s}{4\pi}G\widetilde{G}\right|\eta\rangle = \sqrt{3}f_\pi m_\eta^2\left(F_8\cos\theta - \sqrt{2}F_0\sin\theta\right)$$

$$\langle 0\left|\frac{3\alpha_s}{4\pi}G\widetilde{G}\right|\eta'\rangle = \sqrt{3}f_\pi m_{\eta'}^2\left(F_8\sin\theta + \sqrt{2}F_0\cos\theta\right) \tag{2.11}$$

The next step consists in the evaluation of the matrix elements $\langle 0\,|-2im_s\, \bar{s}\, \gamma_5 s|\,\eta,\eta'\rangle$. Consider

$$\Pi_{\mu\nu}(q^2) = i\int dx e^{iqx}\langle 0\left|TA_\mu^{(\eta)}(x)A_\nu^{(\eta)\dagger}(0)\right|0\rangle = (q_\mu q_\nu - q^2 g_{\mu\nu})\Pi_1(q^2) + q_\mu q_\nu\Pi_2(q^2) \tag{2.12}$$

The Ward-Takahashi identity gives

$$-iq_\mu\Pi_{\mu\nu} = i\int dx e^{iqx}\langle 0\left|T\partial_\mu A_\mu^{(\eta)}(x)A_\nu^{(\eta)\dagger}(0)\right|0\rangle = -iq^2 q_\nu\Pi_0(q^2) \tag{2.13}$$

Defining $T(q^2) = -iq^2\Pi_0(q^2)$ and isolating the η pole

$$T(q^2) = -2i\frac{\langle 0\left|\partial_\mu A_\mu^{(\mu)}\right|\eta\rangle}{(q^2 - m_\eta^2)} + ... \tag{2.14}$$

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In the deep euclidean region the behaviour of $T(q^{2)}$ is known [9]

$$T^{\text{QCD}}(q^2) = \frac{8}{3} \left(\frac{\sqrt{2} \cos \theta}{f_8} + \frac{\sin \theta}{f_0} \right)^2 \frac{\langle 0 \, | -m_s \, \bar{s} \, s | \, 0 \rangle}{q^2} + \dots \tag{2.15}$$

Define $F(t) = tT(t)$ with $t = (q^2 - m_\eta^2)$. $F(t)$ is an analytic function in the complex t plane with a cut starting at $t = (m_\eta + 2m_\pi)^2 - m_\eta^2$ and running along the positive real axis. As a result of Cauchy's theorem then

$$F(0) = -2i \langle 0 \, | \partial_\eta A_\mu^{(\eta)} | \, \eta \rangle = \frac{1}{2\pi i} \int_c \frac{dt}{t} F(t) \tag{2.16}$$

where c is a closed contour consisting of a circle of large radius and two straight lines parallel to the x-axis just above and just below the cut. The main contribution to the integral above is provided by the interval $1.5 \text{GeV}^2 \lesssim q^2 \lesssim 2.5 \text{GeV}^2$ which includes the two pseudoscalar excitations $\eta(1295)$ and $\eta(1440)$. These resonances couple to the η and η' with unknown strengths. On the circle $F(t) \simeq F^{\text{QCD}}(t)$ except for a small region near the real axis. Instead of expression (2.16) consider the following modified integral

$$F(0) = \frac{1}{2\pi i} \int_c dt \left(\frac{1}{t} - a_0 - a_1 t \right) F(t) \tag{2.17}$$

The coefficients a_0 and a_1 are chosen so as to annihilate the integrand at $q^2 = m_1^2 = 1.66$ GeV^2 and at $q^2 = m_2^2 = 2.04 \text{GeV}^2$

$$a_0 = \frac{1}{\mu_1^2} + \frac{1}{\mu_2^2} \quad , \quad a_1 = -\frac{1}{\mu_1^2 \mu_2^2} \quad , \mu_{1,2}^2 = m_{1,2}^2 - m_\eta^2 \tag{2.18}$$

The contribution of the isoscalar pseudoscalar resonances is thus practicaly eliminated and the integrand is reduced to only a few percent of it's initial value over the interval $1.5 \text{GeV}^2 \lesssim q^2 \lesssim 2.5 \text{GeV}^2$.

The main contribution to the integral (2.17) comes now from the circle

$$F(0) \simeq \frac{1}{2\pi i} \oint dt \left(\frac{1}{t} - a_0 - a_1 t \right) F^{\text{QCD}}(t) \tag{2.19}$$

So that using eq.(2.15)

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$$\langle 0 \,|-2im_s\,\bar{s}\,\gamma_5 s|\,\eta\rangle = 2i\sqrt{\frac{2}{3}}\left(\frac{\sqrt{2}\cos\theta}{f_8}+\frac{\sin\theta}{f_0}\right)\left(1+\frac{m_\eta^2}{\mu_1^2}+\frac{m_\eta^2}{\mu_2^2}+\frac{m_\eta^4}{\mu_1^2\mu_2^2}\right)\langle 0 \,|-m_s\,\bar{s}\,s|\,0\rangle$$

$$(2.20)$$

In the equation above the first bracket shows the effect of mixing the second one represents the correction factor due to the isoscalar 0^- continuum, numerically it amounts to 1.431 . In a similar fashion one obtains

$$\langle 0 \,|2im_s\,\bar{s}\,\gamma_5 s|\,\eta'\rangle = 2i\sqrt{\frac{2}{3}}\left(\frac{\cos\theta}{f_0}-\frac{\sqrt{2}\sin\theta}{f_8}\right)\left(1+\frac{m_{\eta'}^2}{\mu_1'^2}+\frac{m_{\eta'}^2}{\mu_2'^2}+\frac{m_{\eta'}^4}{\mu_{1'}^2\mu_{2'}^2}\right)\langle 0 \,|-m_s\,\bar{s}\,s|\,0\rangle$$

$$(2.21)$$

The correction factor due to the continuum now amounts to 4.090 . If the Gell-Mann-Oakes-Renner expression $\langle 0 \,|-m_s\,\bar{s}\,\gamma_5 s|\,0\rangle = f_K^2 m_K^2$ is used, the masses of the η and η' mesons come out

$$m_\eta^2 = \left[.352\left(\frac{\sqrt{2}\cos\theta}{F_8}+\frac{\sin\theta}{F_0}\right)-.212\sin\theta F_8\left(\frac{\cos\theta}{F_0}-\frac{\sqrt{2}\sin\theta}{F_8}\right)\right]\text{GeV}^2 \qquad (2.22)$$

$$m_{\eta'}^2 = \left[1.006\left(\frac{\cos\theta}{F_0}-\frac{\sqrt{2}\sin\theta}{F_8}\right)+.649\cos\theta F_8\left(\frac{\sqrt{2}\cos\theta}{F_8}+\frac{\sin\theta}{F_0}\right)\right]\text{GeV}^2 \qquad (2.23)$$

The unknowns θ ,F_0 and F_8 also appear in the expressions of the widths of the anomaly generated two photon decays of the η and η' [5].

$$\Gamma(\eta\to 2\gamma) = \frac{\alpha^2 m_\eta^3}{192\pi^3 f_\pi^2}\left(\frac{\cos\theta}{F_8}-\frac{2\sqrt{2}\sin\theta}{F_0}\right)^2(1+\Delta_\eta)^2 = (.465\pm.043)\text{keV} \qquad (2.24)$$

$$\Gamma(\eta'\to 2\gamma) = \frac{\alpha^2 m_{\eta'}^3}{192\pi^3 f_\pi^2}\left(\frac{\sin\theta}{F_8}+\frac{2\sqrt{2}\cos\theta}{F_0}\right)^2(1+\Delta_{\eta'})^2 = (4.28\pm.64)\text{keV} \qquad (2.25)$$

$\Delta_{\eta,\eta'}$ result from the chiral symmetry breaking corrections, i.e they represent the deviations from the soft meson limits arising from the large masses of the η and η'. They were evaluated in [4]and found to be quite large, $\Delta_\eta = .77$ and $\Delta_{\eta'} = 6.0$. Numerically, then

$$\frac{\cos\theta}{F_8}-\frac{2\sqrt{2}\sin\theta}{F_0} = .929\pm.094 \qquad (2.26)$$

$$\frac{\sin\theta}{F_8} + \frac{2\sqrt{2}\cos\theta}{F_0} = .308 \pm .050 \tag{2.27}$$

The errors in the equations above represent only the experimental uncertainties in the decay rates. Four equations (2.22), (2.23), (2.24) and (2.25) are now available. The straightforward procedure is to vary θ, use eqs. (2.26) and (2.27) and see what eqs. (2.22) and (2.23) give for the masses. Two limiting cases are considered

$$\theta = -18.5^\circ: \qquad m_\eta = .547\text{GeV} \qquad , \quad m_{\eta'} = 1.011\text{GeV} \tag{2.28}$$

m_η is adjusted to it's experimental value, $m_{\eta'}$deviates by 5.5%.

$$\theta = -30.5^\circ: \qquad m_\eta = .468\text{GeV} \qquad , \quad m_{\eta'} = .958\text{GeV} \tag{2.29}$$

$m_{\eta'}$ is adjusted to it's experimental value, m_η deviates by 14%.

Because of the approximations made (vector meson dominance in obtaining Δ_η and $\Delta_{\eta'}$ in [4] etc.) it is seen that both set of values (2.28) and (2.29) fall well within the expected range. This applies as well to all values of the masses resulting from values of θ between the limits given above. This stability of course forbids any prediction for the exact value of the mixing angle. We shall see that the same applies to all predictions of the present work, i.e. they show a remarkable stability against variations of the mixing angle. The next topic of investigation is the decay $\eta \to 3\pi$.

III. $\eta \to 3\pi$

Consider the neutral mode $\eta \to 3\pi^\circ$. As is well known this decay proceeds through the isospin breaking part of the QCD Hamiltonian

$$H = \frac{1}{2}(m_u - m_d)(\bar{u}\,u - \bar{d}\,d) \tag{3.1}$$

Standard Current -Algebra soft pion techniques yield for the decay rate [2]

$$\Gamma(\eta \to 3\pi^\circ) = \frac{\sqrt{3}}{6912\pi^2}\delta_\eta \frac{(m_\eta - 3m_\pi)^2}{m_\eta} \frac{(m_d - m_u)^2}{f_\pi^6} |\langle 0|\bar{u}\,\gamma_5 u|\eta\rangle|^2 \tag{3.2}$$

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$\delta_\eta = .86$ is a kinematical factor.

The matrix element $\langle 0\,|\bar{u}\,\gamma_5 u|\,\eta\rangle$ can be evaluated in exactly the same way $\langle 0\,|\bar{s}\,\gamma_5 s|\,\eta\rangle$ was with the result

$$\langle 0\,|\bar{u}\,\gamma_5 u|\,\eta\rangle = \frac{-1}{\sqrt{3}f_\pi}\left(\frac{\cos\theta}{F_8} - \frac{\sqrt{2}\sin\theta}{F_0}\right)\left(1 + \frac{m_\eta^2}{\mu_1^2} + \frac{m_\eta^2}{\mu_2^2} + \frac{m_\eta^4}{\mu_1^2\mu_2^2}\right)\langle 0\,|\bar{u}\,u|\,0\rangle \qquad (3.3)$$

Note that the continuum enhancement factor appearing in the second bracket is the same as in eq.(2.20) as it arises from the same isoscalar 0^- continuum . The Gell-Mann-Oakes -Renner expression $\langle 0\,|-(m_u + m_d)\,\bar{u}\,u|\,0\rangle = f_\pi^2 m_\pi^2$ gives then for the decay rate

$$\Gamma(\eta \to 3\pi^\circ) = \frac{1.431^2}{\sqrt{3}6912\pi^2}\left(\frac{\cos\theta}{F_8} - \frac{\sqrt{2}\sin\theta}{F_0}\right)^2 \delta_\eta \frac{(m_\eta - 3m_\pi)^2}{m_\eta}\left(\frac{m_d - m_u}{m_d + m_u}\right)^2 \frac{m_\pi^4}{f_\pi^4}(1 + C)$$

$$(3.4)$$

C is a rescattering enhancement factor which was estimated to be .83 by Roiesnel and Truong [6]. It was argued however by Gasser and Leutwyler [2] that one should take instead $C \simeq .50$ a value we shall here use. Equation (3.4) can now be used to evaluate the ratio $\frac{m_u}{m_d}$ of the quark masses. For $\theta = -18.5^\circ$ one gets $\frac{m_u}{m_d} = .47$. This ratio decreases to $\frac{m_u}{m_d} = .42$ for $\theta = -30.5^\circ$.

IV. RADIATIVE VECTOR-MESON DECAYS

The radiative decays of vector-mesons involving η or η' were studied in [2] using the hypothesis of vector-meson dominance to relate them to the two photon decays of η and η'. The only improvement we can provide is to multiply the expressions of Ball et al. [2] for the coupling constants by the chiral symmetry breaking enhancement factors $(1 + \Delta_{\eta,\eta'})$, thus

$$g_{\rho\eta\gamma} = \frac{1}{4\pi}\sqrt{\frac{3}{2}}\frac{m_\rho}{f_\rho\pi^2}\left(\frac{\cos\theta}{F_8} - \frac{\sqrt{2}\sin\theta}{F_0}\right)(1 + \Delta_\eta) \qquad (4.1)$$

$$g_{\rho\eta'\gamma} = \frac{1}{4\pi}\sqrt{\frac{3}{2}}\frac{m_\rho}{f_\rho\pi^2}\left(\frac{\sin\theta}{F_8} + \frac{\sqrt{2}\cos\theta}{F_0}\right)(1 + \Delta_{\eta'}) \qquad (4.2)$$

etc.

Decays involving ω and ϕ vector-mesons suffer from an additional uncertainty in the $\omega - \phi$ mixing angle to which the decay rates might be very sensitive. The results for $g_{VP\gamma}$ in GeV^{-1} are listed in table 1

	$g_{\rho\eta\gamma}$	$g_{\rho\eta'\gamma}$	$g_{\omega\eta\gamma}$	$g_{\omega\eta'\gamma}$	$g_{\phi\eta\gamma}$	$g_{\phi\eta'\gamma}$
Experiment	$1.85 \pm .35$	$\lesssim 1.31$	$.60 \pm .15$	$.45 \pm .06$	$.70 \pm .03$	$.73 \pm .16$
$\theta = -18.5°$	1.57	.59	.52	.56	1.06	2.43
$\theta = -30.5°$	1.38	.35	.50	.52	.68	2.93

<div align="center">Table 1</div>

Once again the results are quite stable against variations of the mixing angle and agreement with experiment is reasonnably good except for $g_{\phi\eta'\gamma}$.

An additional item of investigation is the ratio $r = \frac{\langle 0|G\widetilde{G}|\eta'\rangle}{\langle 0|G\widetilde{G}|\eta\rangle}$ for which we get

$$r = 5.1 \qquad for \qquad \theta = -18.5° \tag{4.3}$$

and

$$r = 2.64 \qquad for \qquad \theta = -30.5° \tag{4.4}$$

Novikov et al. [8], assuming that the decay mechanism for $J/\Psi \to \eta(\eta')\gamma$ via the strong anomaly dominates in addition to vector meson dominance in the J/Ψ channel, relate r to the ratio of the corresponding decay rates, they give

$$r = \sqrt{\frac{\Gamma(J/\Psi \to \eta'\gamma)}{\Gamma(J/\Psi \to \eta\gamma)}} \left(\frac{1 - \frac{m_\eta^2}{m_{J/\Psi}^2}}{1 - -\frac{m_{\eta'}^2}{m_{J/\Psi}^2}} \right)^{\frac{3}{2}} = 2.48 \pm .15 \tag{4.5}$$

which is reasonnably consistent with our values.

V. CONCLUSION

The effect of the contribution of the isoscalar pseudoscalar continuum on the glue content and mixing angle of the $\eta - \eta'$ system. The masses of the isoscalar mesons and their decay rates into two photons are consistently described in terms of a single octet-singlet mixing

angle θ and their values are quite stable against variations of θ. Applications to the decays $\eta \to 3\pi$ and $V \to P\gamma$ were also considered. Agreement with experiment is in general quite good.

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